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AUTOCORRELATION-BASED SPECTRUM SENSING ALGORITHMS FOR COGNITIVE RADIOS (POSTPRINT)

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Autocorrelation-Based Spectrum Sensing Algorithms for Cognitive Radios

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Abstract—Cognitive radio is an enabling technology for opportunistic spectrum access. Spectrum sensing is a key feature of a cognitive radio whereby a secondary user can identify and utilize the spectrum that remains unused by the licensed (primary) users. Among the recently proposed algorithms the covariance-based method of [1] is a constant false alarm rate (CFAR) detector with a fairly low computational complexity. The low computational complexity reduces the detection time and improves the radio agility. In this paper, we present a framework to analyze the performance of this covariance-based method. We also propose a new spectrum sensing technique based on the sample autocorrelation of the received signal. The performance of this algorithm is also evaluated through analysis and simulation. The results obtained from simulation and analysis are very close and verify the accuracy of the approximation assumptions in our analysis. Furthermore, our results show that our proposed algorithm outperforms the algorithm in [1].

Index Terms—Spectrum sensing, Dynamic Spectrum Access, Cognitive Radio, Autocorrelation-based

I. INTRODUCTION

In 2002, Federal Communications Commission (FCC) Spectral Policy Task Force reported that typical radio channel occupancy is less than 15% while the peak occupancy is close to 85% [2]. To increase the spectrum usage efficiency, the task force recommended the development of opportunistic or dynamically spectrum access. Dynamic spectrum access meshes well with the cognitive radio [3] paradigm which, as an evolution of software-defined radios, is aware of its surrounding environment and can accordingly adapt its internal operating states [4].

Dynamic spectrum access requires frequency agile radios that can monitor and identify the spectral bands that are unused by the primary licensed users (the so-called white spaces). The radio will then dynamically adjust its carrier frequency, transmit power, modulation, coding, etc, in order to make best use of the available spectrum and achieve the desired quality of service. In order to avoid interference to primary users, spectrum sensing must detect signals with low probability of detection. Moreover, low computational complexity and ease of implementation is required in order to facilitate radio agility.

Akyildiz et al. categorized non-cooperative spectrum sensing into three categories [5]: energy detection, matched-filter detection, and cyclostationary feature detection. Spectrum sensing using energy detection is easy to implement and

performs well if the noise power at the receiver is known. However, uncertainty in the noise power can significantly degrade the performance of energy detectors. Furthermore, estimation of the noise power which is required in such cases leads to the so called “SNR wall” phenomena [6]. The detectors based on matched filtering and cyclostationary features, on the other hand, rely on the a priori knowledge of the signal parameters. The matched-filter based system is a coherent system and must have a priori knowledge of the modulation type and carrier frequency of the primary user. The cyclostationary feature detectors also require some parameters of the primary signal such as symbol rate and have high computational requirements.

An alternative approach is autocorrelation-based method suggested in [1]. Their approach relies on the fact that when the receiver bandwidth is greater than the bandwidth of the signal, the autocorrelation function of signal plus noise is distinguishable from that of noise alone (which is assumed to be white). Because autocorrelation function is a one-dimensional entity as opposed to the two-dimensional cyclic autocorrelation function, the burden in computing the decision statistics is drastically reduced. The decision statistic in [1] is based on the Frobenious norm of the correlation matrix of the received signal. They, however, have not analyzed the performance of their detector and only presented an empirical false alarm rate.

In this paper, we present an accurate analysis of the autocorrelation-based detector in [1] using the results in [7] on the first-order autoregressive (AR) spectral estimator. The presented analysis can be extended for the performance of other autocorrelation-coefficient based algorithms. We also propose a new autocorrelation-based detector by considering additional properties of the autocorrelation function of typical baseband communication signals. These properties greatly simplify the complexity of the detection statistics resulting in a linear detector as opposed to the quadratic form in [1]. We also evaluate the performance of our detector through analysis and simulation.

The remainder of this paper is organized as follows. In Section II we present the spectrum sensing problem, the system configuration and the assumptions on the received signal. Section III introduces the detector in [1] as well as our new

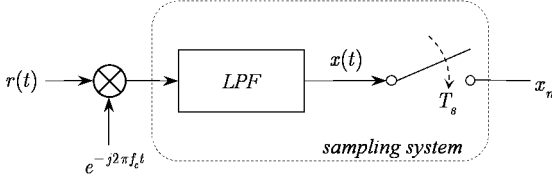


Fig. 1. Simplified block diagram of ideal RF front-end.

detector. Section IV presents the analysis of the performance of detectors in terms of detection and false alarm probabilities. Section V presents numerical results from simulation and analysis. Finally, concluding remarks are provided in Section VII.

II. SPECTRUM SENSING SYSTEM

For the purpose of spectrum sensing the cognitive radio front-end can be simplified as shown as in Fig. 1. The radio receives an RF signal $r(t)$ and after down conversion, low-pass filtering, and sampling, obtains the complex baseband signal $\{x_n\}$. The target center frequency is f_c Hz, the bandwidth of the low-pass filter is $(-f_{bw}, f_{bw})$ Hz, and the sampling rate T_s is given by $T_s \triangleq (2f_{bw})^{-1}$.

The complex baseband signal $x(t)$ is modeled as $x(t) \triangleq \eta s(t)e^{j(2\pi\tilde{f}_c t + \theta_0)} + n(t)$ where $s(t)$ is the primary baseband communication signal, $n(t)$ is the complex noise process, and where \tilde{f}_c and θ_0 denote the frequency and phase offsets of the local oscillator from that of the primary transmitter, respectively. The value of $\eta \in \{0, 1\}$ determines the presence or absence of the primary signal $s(t)$. Therefore the detection of the primary signal is described by the following binary hypotheses testing problem.

$$\begin{aligned} H_0 : \quad & \eta = 0, \quad \text{primary signal absent} \\ H_1 : \quad & \eta = 1, \quad \text{primary signal present} \end{aligned} \quad (1)$$

The primary signal $s(t)$ is unknown and is modeled as a complex-valued zero-mean wide-sense stationary (WSS) process, characterized by its autocorrelation function $r_s(\tau) \triangleq E[s(t)s(t-\tau)^*]$. Furthermore, $s(t)$ is band-limited in the frequency range $(-f_b, f_b)$ Hz where $f_b < f_{bw}$.

The spectrum sensing algorithm processes the complex baseband signal $x_n \triangleq x(nT_s)$. Like $x(t)$, x_n can be separated into two components:

$$x_n = \eta s_n e^{j\omega_0 n + \theta_0} + n_n \quad (2)$$

where $s_n \triangleq s(nT_s)$, $n_n \triangleq n(nT_s)$, and $\omega_0 \triangleq 2\pi\tilde{f}_c T_s$. The autocorrelation function of s_n is denoted by $r_{s,l} \triangleq r_s(lT_s)$. We note that the condition $(f_b < f_{bw})$ guarantees that s_n is non-white, i.e., $r_{s,l} \neq r_{s,0}\delta_l$ where δ_l is the Kronecker delta function.

The complex-valued noise component, $\{n_n\}$, is modeled as a circular white Gaussian noise process with mean zero and variance σ_n^2 . Therefore, the autocorrelation function of the noise process is given by $r_{n,l} = \sigma_n^2\delta_l$. Accordingly, the

signal-to-noise ratio (SNR) of x_n is denoted by

$$\gamma \triangleq \frac{r_{s,0}}{\sigma_n^2}. \quad (3)$$

Assuming that $\{s_n\}$ and $\{n_n\}$ are uncorrelated, we express the conditional autocorrelation function of x_n as

$$r_{l|H_\eta} = \eta r_{s,l} e^{j\omega_0 l} + \sigma_n^2 \delta_l \quad (4)$$

It is further assume that the real and imaginary parts of $\{s_n\}$, namely $\{\Re(s_n)\}$ and $\{\Im(s_n)\}$ are independent. The following lemma results.

Lemma 1: If the real and imaginary components of a complex-valued zero-mean WSS random process are independent, then the autocorrelation function of the process is an even function.

Proof: Let a complex-valued zero-mean WSS process $x_n \triangleq y_n + jz_n$ where y_n and z_n are mutually independent real WSS processes. Then,

$$\begin{aligned} r_{xx,l} &= E[x_n x_{n-l}^*] \\ &= E[(y_n + jz_n)(y_{n-l} - jz_{n-l})] \end{aligned} \quad (5)$$

where $*$ denotes complex conjugation. Since x_n is zero-mean and y_n and z_n are independent, we have

$$\begin{aligned} r_{xx,l} &= E[y_n y_{n-l}^*] E[z_n z_{n-l}^*] \\ &\triangleq r_{yy,l} + r_{zz,l} \end{aligned} \quad (6)$$

Since y_n and z_n are real processes, their autocorrelation functions $r_{yy,l}$ and $r_{zz,l}$ are both real and even. Thus, $r_{xx,l}$ is also real and even. ■

Lemma 2: Let x_n be a lowpass WSS process with real and even autocorrelation function $r_{xx,l}$ and cutoff frequency $\omega_c \in (0, \pi/2)$. Then, there exists a non-negative integer N_c such that

$$r_{xx,l} > 0 \quad \text{for all } |l| < N_c \quad (7)$$

If the process is an ideal lowpass process, then $N_c = \lceil \pi/\omega_c \rceil$.

Proof: By the property of the discrete-time Fourier transform, the PSD of x_n is real and even. Let the PSD of x_n be defined as

$$P_{xx}(\omega) = \begin{cases} f(\omega), & 0 \leq \omega < \omega_c \\ f(-\omega), & -\omega_c \leq \omega < 0 \\ 0, & \text{o.w.} \end{cases} \quad (8)$$

where $f(\omega)$ is a positive real function. Then,

$$r_{xx,l} = \frac{1}{\pi} \int_0^{\omega_c} f(\omega) \cos(\omega l) d\omega \quad (9)$$

We observe that $f(\omega) \cos(\omega l) > 0$ for all $\omega \in (0, \pi/(2l))$. Thus, $N_c = \lceil \pi/(2\omega_c) \rceil$ satisfies (7). Furthermore, if the process is an ideal “brick” lowpass process, i.e., $f(\omega) = 1$, then

$$\begin{aligned} r_{xx,l} &= \frac{1}{\pi} \int_0^{\omega_c} \cos(\omega l) d\omega \\ &= \frac{\sin(\omega_c l)}{\pi l} \end{aligned} \quad (10)$$

Therefore, $N_c = \lceil \pi/\omega_c \rceil$ satisfies (7). ■

III. SPECTRUM SENSING ALGORITHMS

Taking N samples of x_n , i.e., $\mathbf{x} = (x_0, x_1, \dots, x_{N-1})$, a spectrum sensing algorithm forms a decision statistic $T(\mathbf{x})$ and compares it to threshold λ , i.e.,

$$T(\mathbf{x}) \begin{cases} < \lambda & \text{decide } H_0 \\ > \lambda & \text{decide } H_1. \end{cases} \quad (11)$$

The decision statistics in [1] as well as our proposed detector are based on the estimates of autocorrelation of \mathbf{x} .

$$\hat{r}_l \triangleq \begin{cases} \frac{1}{N-l} \sum_{n=0}^{N-l-1} x_{n+l} x_n^*, & l \geq 0 \\ -\hat{r}_l^*, & l < 0 \end{cases} \quad (12)$$

We note that \hat{r}_l is an unbiased and consistent estimator of $r_{xx,l}$. For ease of notation, in the following we will drop the dependence of the decision statistic on the sample data \mathbf{x} .

A. Zeng-Liang signal detector

Zeng and Liang, [1], proposed a spectrum-sensing technique base on the following decision statistic,

$$\sum_{l=-L}^L \left(1 - \frac{|l|}{L+1}\right) |\hat{r}_l|^2 \leq \lambda_{ZL} \hat{r}_0^2 \quad (13)$$

where the parameter L is chosen so that the magnitude of the signal autocorrelation function $|r_{s,l}|$ is significant for all $|l| < L$. The weighting scheme of $|\hat{r}_l|^2$ on the left-hand side of (13) is formed as the left-hand side is derived from the Frobenius norm of estimate of the covariance matrix. Equivalently, (13) can be reformulated to fit (11) as follows.

$$\tilde{T}_{ZL} \triangleq \sum_{l=-L}^L w_l \frac{|\hat{r}_l|^2}{\hat{r}_0^2} \quad (14)$$

with weighting function

$$w_l \triangleq \frac{L+1-|l|}{L+1} \quad (15)$$

Because the autocorrelation function is conjugate symmetric and the $(l = 0)$ term is always 1, (without affecting the performance), the decision statistic in (14) can be simplified to

$$T_{ZL} \triangleq \sum_{l=1}^L w_l \frac{|\hat{r}_l|^2}{\hat{r}_0^2} \quad (16)$$

B. A new correlation-based detector

The Zeng-Liang detector is designed to account for the nonwhite nature of the (oversampled) primary communication signal. By further incorporating the assumption that the primary signal s_n is (A1) lowpass and (A2) complex-valued with independent real and imaginary components, we can improve the performance of the autocorrelation-based detector.

By (A2) and Lemma 1, $r_{l|H_1} e^{-j\omega_0 l}$ is a real-valued function. Moreover, introducing (A2) and Lemma 2 reveal that there exists an integer $N_c > 1$ such that $r_{s,l}$ is real and strictly positive for all $l \in (-N_c, N_c)$. Hence, under H_1 , $\Re\{r_{l|H_1} e^{-j\omega_0 l}\} > 0$ for all $l \in (-N_c, N_c)$ while

$\Im\{r_{l|H_1} e^{-j\omega_0 l}\} = 0$ for all l . On the other hand, under H_0 , $r_{l|H_0} e^{-j\omega_0 l} = 0$ for all $l \neq 0$.

In general, a decision statistic is designed so that its conditional means under the two hypotheses are different. The distinguishable feature in $r_{l|H_1}$ is in the real part while the imaginary part is zero under both hypotheses. Hence, we form the decision statistic for our proposed detector to be

$$T_{IN}(\omega) \triangleq \sum_{l=1}^L w_l \frac{\Re\{\hat{r}_l e^{-j\omega l}\}}{\hat{r}_0} \quad (17)$$

Scaling by \hat{r}_0 results in a constant false-alarm rate (CFAR) detector. The limit L should be chosen so that $r_{l|H_1} > 0$ for all $L \leq N_c$. The frequency scanning parameter ω enables the algorithm to scan across the frequency band in order to locate the center frequency of the captured primary signal. While weighting coefficients w_l can be optimized, in this paper we have opted to use (15) for their value.

IV. PERFORMANCE ANALYSIS

In this section, the performance of the two spectrum sensing algorithms presented in the previous section is evaluated analytically. To this end, we first determine the statistical distributions of the autocorrelation estimates \hat{r}_l . Then, the performance of the Zeng-Liang detector can be assessed following the procedures used by Kay [7] to analyze the performance of an autoregressive detector. The performance of our proposed detector is also analyzed using the same approach. The analysis does not make use of either (A1) or (A2) and only assumes that $\{s_n\}$ is a generic complex-valued zero-mean WSS process with autocorrelation function $r_{s,l}$.

A. Asymptotic conditional distributions of \hat{r}_l

In this section, we present the probability density function (pdf) of the autocorrelation estimates \hat{r}_l in (12). Kay [7] has investigated this problem for the case of an autoregressive AR(1) process with a fixed noise power. Here, following his approach, we present a generalization to his results, accounting for an arbitrary SNR value.

To compactly formulate the statistics, we analyze the scaled version of \hat{r}_l , namely

$$\hat{y}_l = \frac{\hat{r}_l}{\sigma_n^2} \quad (18)$$

We note that \hat{y}_l can replace \hat{r}_l in (16) and (17) without affecting the detection statistics. Let $\hat{y}_l = \hat{\alpha}_l + j\hat{\beta}_l$. Under each hypothesis (H_0 or H_1) and for large N , $\hat{\alpha}_l$ and $\hat{\beta}_l$ can be shown to be jointly Gaussian distributed by the central limit theorem. Consequently, finding the conditional mean and conditional (co)variances of $\hat{\alpha}_l$ and $\hat{\beta}_l$ completely determines their statistics. The conditional mean under each hypothesis is found as follows.

$$E[\hat{\alpha}_l | H_\eta] = \delta_l + \eta \gamma \rho_{r,l} \quad (19)$$

and

$$E[\hat{\beta}_l | H_\eta] = \eta \gamma \rho_{i,l} \quad (20)$$

where $\rho_{r,l} \triangleq \Re\{r_{s,l}e^{j\omega_0 l}\}/r_{s,0}$ and $\rho_{i,l} \triangleq \Im\{r_{s,l}e^{j\omega_0 l}\}/r_{s,0}$. Asymptotically as $N \rightarrow \infty$ the conditional covariances of $\hat{\alpha}_l$ and $\hat{\beta}_l$ are evaluated as

$$\begin{aligned} \text{cov}[\hat{\alpha}_l, \hat{\alpha}_m | H_\eta] &= \frac{1}{2N}(\delta_{l-m} + \delta_l \delta_m) + \eta \frac{\gamma}{N}(\rho_{r,l-m} + \rho_{r,l+m}), \quad (21) \end{aligned}$$

$$\begin{aligned} \text{cov}[\hat{\beta}_l, \hat{\beta}_m | H_\eta] &= \frac{1}{2N}(\delta_{l-m} - \delta_l \delta_m) + \eta \frac{\gamma}{N}(\rho_{r,l-m} - \rho_{r,l+m}), \quad (22) \end{aligned}$$

and

$$\text{cov}[\hat{\alpha}_l, \hat{\beta}_m | H_\eta] = \eta \frac{\gamma}{N}(\rho_{i,l-m} + \rho_{i,l+m}). \quad (23)$$

B. Zeng-Liang detector, T_{ZL}

Zeng and Liang [1] only provided an empirical formulation for the probability of false alarm from simulations of their detector and did not provide the probability of detection. Using an approach similar to that in [7] we evaluate the cumulative distribution function (cdf) of the decision statistic from the characteristic function of a related random variable [8].

First, we note that (16) can be written as

$$T_{ZL} = \sum_{l=1}^L w_l \frac{|\hat{\alpha}_l|^2 + |\hat{\beta}_l|^2}{|\hat{\alpha}_0|^2} \quad (24)$$

Furthermore, we observe that the cdf of T_{ZL} can be written as

$$\begin{aligned} F_{T_{ZL}}(t) &= \Pr \left\{ \sum_{l=1}^L w_l |\hat{\alpha}_l|^2 + \sum_{l=1}^L w_l |\hat{\beta}_l|^2 - t |\hat{\alpha}_0|^2 < 0 \right\} \quad (25) \end{aligned}$$

Let

$$S_{ZL}(t) \triangleq \mathbf{y}^T \mathbf{W}(t) \mathbf{y} \quad (26)$$

where $()^T$ denotes matrix transpose,

$$\mathbf{y} \triangleq [\hat{\alpha}_0 \quad \hat{\alpha}_1 \quad \cdots \quad \hat{\alpha}_L \quad \hat{\beta}_1 \quad \cdots \quad \hat{\beta}_L]^T \quad (27)$$

and

$$\mathbf{W}(t) = \text{diag}([-t \quad w_1 \quad \cdots \quad w_L \quad w_1 \quad \cdots \quad w_L]). \quad (28)$$

Then, the cdf of T_{ZL} can be expressed in terms of the cdf of $S_{ZL}(t)$.

$$\begin{aligned} F_{T_{ZL}}(t) &= \Pr\{S_{ZL}(t) < 0\} \\ &= F_{S_{ZL}(t)}(0) \end{aligned} \quad (29)$$

where $F_{S_{ZL}(t)}(s)$ is the cdf of $S_{ZL}(t)$. Hence, the cdf of T_{ZL} can be evaluated utilizing the characteristic function $\phi_{S_{ZL}}(\omega)$ of $S_{ZL}(t)$. For a random variable X with characteristic function $\phi_X(\omega)$, its cdf can be computed from [8]

$$\begin{aligned} F_X(x) &= \frac{1}{2} - \frac{1}{\pi} \\ &\times \int_0^\infty \frac{\text{Im}[\phi_X(\omega)] \cos \omega x - \text{Re}[\phi_X(\omega)] \sin \omega x}{\omega} d\omega. \quad (30) \end{aligned}$$

Applying this characteristic function property to (29), we have

$$\begin{aligned} F_{T_{ZL}}(t) &= F_{S_{ZL}(t)}(0) \\ &= \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{\text{Im}[\phi_{S_{ZL}(t)}(\omega)]}{\omega} d\omega \quad (31) \end{aligned}$$

Assuming that N is large, $\hat{\alpha}_l$ and $\hat{\beta}_l$ are jointly Gaussian distributed under both hypothesis. Hence, the random vector \mathbf{y} is a Gaussian random vector. Denoting $\bar{\mathbf{y}}_\eta$ and \mathbf{K}_η to be the conditional mean and covariance matrix, respectively, of \mathbf{y} , the (conditional) characteristic function of a quadratic form, $\mathbf{y}^T \mathbf{W} \mathbf{y}$, of Gaussian vector is [9]

$$\begin{aligned} \phi_{z|H_\eta}(\omega) &= \frac{1}{|\mathbf{I} - 2j\omega \mathbf{K}_\eta \mathbf{W}|^{1/2}} \\ &\times \exp \left[j\omega \bar{\mathbf{y}}_\eta^T \mathbf{W} (\mathbf{I} - 2j\omega \mathbf{K}_\eta \mathbf{W})^{-1} \bar{\mathbf{y}}_\eta \right] \quad (32) \end{aligned}$$

We note that there is no known closed-form solution to the resulting integral, so (31) needs to be numerically evaluated.

Finally, we can formulate the probabilities of false alarm and detection in terms of $\phi_{z|H_\eta}(\omega)$. Under the null hypothesis H_0 , the probability of false alarm given the threshold λ_{ZL} is computed by

$$\begin{aligned} P_{fa,ZL} &= 1 - F_{T_{ZL}|H_0}(\lambda_{ZL}) \\ &= \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \frac{\text{Im}\{\phi_{S_{ZL}|H_0}(\omega)\}}{\omega} d\omega \quad (33) \end{aligned}$$

Similarly, under alternate hypothesis H_1 , the probability of detection given the threshold λ_{ZL} is computed by

$$\begin{aligned} P_{d,ZL} &= 1 - F_{T_{ZL}|H_1}(\lambda_{ZL}) \\ &= \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \frac{\text{Im}\{\phi_{S_{ZL}|H_1}(\omega)\}}{\omega} d\omega \quad (34) \end{aligned}$$

Based on (19)-(23), the statistics of \mathbf{y} to evaluate (32) can be expressed as follows. The conditional mean of \mathbf{y} is found to be

$$\begin{aligned} \bar{\mathbf{y}}_\eta &= [1 \quad 0 \quad \cdots \quad 0]^T \\ &+ \eta [1 \quad \rho_{r,1} \quad \cdots \quad \rho_{r,L} \quad \rho_{i,1} \quad \cdots \quad \rho_{i,L}]^T, \quad (35) \end{aligned}$$

and the conditional covariance matrix of \mathbf{y} is found to be

$$\mathbf{K}_\eta = \frac{1}{2N} \text{diag}([2 \quad 1 \quad \cdots \quad 1]) + \eta \frac{\gamma}{N} \begin{bmatrix} \mathbf{Q}_1 & \mathbf{Q}_2 \\ \mathbf{Q}_2^T & \mathbf{Q}_3 \end{bmatrix}. \quad (36)$$

The submatrices \mathbf{Q}_1 , \mathbf{Q}_2 , and \mathbf{Q}_3 in (36) are composed of a Toeplitz matrix and a Hankel matrix.

$$\mathbf{Q}_1 = \mathcal{T}(\mathbf{q}_{r,0,L}, \mathbf{q}_{r,0,L}) + \mathcal{H}(\mathbf{q}_{r,0,L}, \mathbf{q}_{r,L,2L}), \quad (37)$$

$$\begin{aligned} \mathbf{Q}_2 &= \mathcal{T}(\mathbf{q}_{i,-1,L-1}, \mathbf{q}_{i,-1,L-1}) \\ &+ \mathcal{H}(\mathbf{q}_{i,1,L+1}, \mathbf{q}_{i,L+1,2L}), \quad (38) \end{aligned}$$

and

$$\begin{aligned} \mathbf{Q}_3 &= \mathcal{T}(\mathbf{q}_{r,0,L-1}, \mathbf{q}_{r,0,L-1}) \\ &- \mathcal{H}(\mathbf{q}_{r,2,L+1}, \mathbf{q}_{r,L+1,2L}), \quad (39) \end{aligned}$$

where $\mathcal{T}(\mathbf{c}, \mathbf{r})$ represents a Toeplitz matrix with the first column \mathbf{c} and the first row \mathbf{r} , $\mathcal{H}(\mathbf{c}, \mathbf{r})$ represents a Hankel matrix with the first column \mathbf{c} and the last row \mathbf{r} . Furthermore,

$$\mathbf{q}_{r,a,b} \triangleq [\rho_{r,a} \quad \rho_{r,a+1} \quad \cdots \quad \rho_{r,b}]^T \quad (40)$$

and

$$\mathbf{q}_{i,a,b} \triangleq [\rho_{i,a} \quad \rho_{i,a+1} \quad \cdots \quad \rho_{i,b}]^T \quad (41)$$

C. The proposed detector, T_{IN}

To follow similar steps as the Zeng-Liang algorithm, we first rewrite the proposed decision statistic in (17) in terms of $\hat{\alpha}_l$ and $\hat{\beta}_l$ by

$$T_{IN}(\omega) = \frac{\sum_{l=1}^L w_l [\hat{\alpha}_l \cos(\omega l) + \hat{\beta}_l \sin(\omega l)]}{\hat{\alpha}_0} \quad (42)$$

Let

$$S_{IN}(t, \omega) \triangleq \mathbf{w}^T(t) \mathbf{z}(\omega) \quad (43)$$

where

$$\mathbf{z}(\omega) \triangleq [\hat{\alpha}_0 \quad \hat{\alpha}_1 \cos(\omega) + \hat{\beta}_1 \sin(\omega) \quad \cdots \quad \hat{\alpha}_L \cos(\omega L) + \hat{\beta}_L \sin(\omega L)]^T \quad (44)$$

and

$$\mathbf{w}(t) = [-t \quad w_1 \quad \cdots \quad w_L]^T. \quad (45)$$

Then, the cdf of $T_{IN}(\omega)$ can be written as

$$F_{T_{IN}}(t, \omega) = \Pr\{S_{IN}(t, \omega) < 0\} \\ = F_{S_{IN}(t, \omega)}(0) \quad (46)$$

Similar to the random vector \mathbf{y} in (27), the random vector \mathbf{z} is Gaussian under both hypothesis. Hence, $S_{IN}(t)$ in (43) is also Gaussian and the conditional cdf of T_{IN} is given by

$$F_{T_{IN}|H_\eta}(t, \omega) \triangleq Q\left(\frac{\mathbf{w}^T(t) \bar{\mathbf{z}}_\eta(\omega)}{\sqrt{\mathbf{w}^T(t) \mathbf{C}_\eta(\omega) \mathbf{w}(t)}}\right) \quad (47)$$

where $\bar{\mathbf{z}}_\eta$ is the conditional mean of \mathbf{z} , \mathbf{C}_η is the conditional covariance matrix of \mathbf{z} , and $Q(x)$ is the Q-function. Accordingly, given the threshold λ_{IN} , the probability of false alarm is determined to be

$$P_{fa, IN}(\omega) = 1 - F_{T_{IN}|H_0}(\lambda_{IN}, \omega) \\ = Q\left(\frac{-\mathbf{w}^T(\lambda_{IN}) \bar{\mathbf{z}}_0(\omega)}{\sqrt{\mathbf{w}^T(\lambda_{IN}) \mathbf{C}_0(\omega) \mathbf{w}(\lambda_{IN})}}\right) \quad (48)$$

Similarly, the probability of detection can be evaluated by

$$P_{d, IN}(\omega) = 1 - F_{T_{IN}|H_1}(\lambda_{IN}, \omega) \\ = Q\left(\frac{-\mathbf{w}^T(\lambda_{IN}) \bar{\mathbf{z}}_1(\omega)}{\sqrt{\mathbf{w}^T(\lambda_{IN}) \mathbf{C}_1(\omega) \mathbf{w}(\lambda_{IN})}}\right) \quad (49)$$

The conditional statistics of $\mathbf{z}(\omega)$ are defined as follows. Under H_η , the conditional mean $\bar{\mathbf{z}}_\eta(\omega)$ is found to be

$$\bar{\mathbf{z}}_\eta(\omega) = [1 \quad 0 \quad \cdots \quad 0]^T \\ + \eta [1 \quad \rho_1(\omega) \quad \cdots \quad \rho_L(\omega)]^T \quad (50)$$

with

$$\rho_l(\omega) \triangleq \rho_{r,l} \cos(\omega l) + \rho_{i,l} \sin(\omega l) \quad (51)$$

The conditional covariance matrix $\mathbf{C}_\eta(\omega)$ is determined by

$$\mathbf{C}_\eta(\omega) = \frac{1}{2N} \text{diag}([2 \quad 1 \quad \cdots \quad 1]^T) + \eta \frac{\gamma}{N} \\ \times [\mathcal{T}\{\mathbf{p}_{0,L}(\omega), \mathbf{p}_{0,L}(\omega)\} + \mathcal{H}\{\mathbf{p}_{0,L}(\omega), \mathbf{p}_{L,2L}(\omega)\}] \quad (52)$$

where

$$\mathbf{p}_{a,b}(\omega) \triangleq [\rho_a(\omega) \quad \rho_{a+1}(\omega) \quad \cdots \quad \rho_b(\omega)]^T. \quad (53)$$

The expression for the probability of false alarm in (48) can be further simplified because \mathbf{C}_0 is diagonal;

$$P_{fa, IN} = Q\left(\lambda_{IN} \left[\frac{\lambda_{IN}^2}{N} + \frac{1}{2N} \sum_{i=1}^L w_i^2\right]^{-\frac{1}{2}}\right). \quad (54)$$

The false-alarm rate $P_{fa, IN}$ is independent of ω . It is readily observed from (54) that the probability of false alarm does not depend on any signal parameters.

V. NUMERICAL RESULTS

In this section, the detector performance that we obtained in the previous section is verified against Monte-Carlo simulation results. Following common configurations are used throughout the section. All detectors use the same number of samples, namely $N = 1000$. The detectors are configured with $L = 2$. For both detectors the weighting function w_l is as defined in (15).

The received signal contains only one communication signal under H_1 case. The primary signal under detection is 16QAM signal with rectangular pulse shaping, transmitted over an AWGN channel. The transmitted symbols are drawn randomly with equal probabilities among all possible symbols. The detectors oversample the received signal at $N_s = 3$ samples/symbol (i.e., $f_{bw} = 3f_b$). Hence, the autocorrelation function of s_n is given by

$$r_{s,l} = r_{s,0} \begin{cases} \frac{N_s - |l|}{N_s}, & |l| < N_s \\ 0, & \text{o.w.} \end{cases} \quad (55)$$

In the simulations, the performances are evaluated over 10000 independent trials for each setup. To account for the phase offset θ_0 in (2), θ_0 is randomly drawn from $[0, 2\pi)$ for each trial.

Fig. 2 illustrates the receiver operating characteristic (ROC) curve of the two detectors under fixed $\gamma = -12$ dB, and Fig. 3 shows the detection probabilities as functions of the SNR while the false-alarm rate is fixed to a constant value of 0.01. For both cases, $\omega_0 = 0$ and T_{IN} is evaluated with $\omega = \omega_0 = 0$. The figures illustrate the improved performance of the proposed algorithm over that in [1]. Furthermore, the theoretical results are in a very good agreement with the results from simulation.

Next, we observe the sensitivity of the proposed algorithm to its scanning frequency offset. For this example, we fix $P_{fa} = 0.01$ and $\gamma = -8$ dB and introduce primary carrier

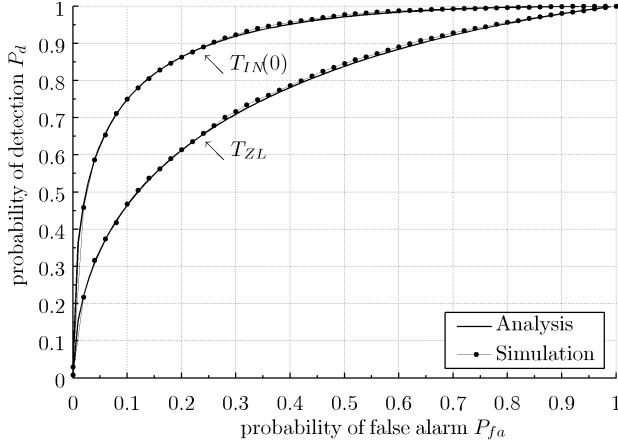


Fig. 2. ROC curves of T_{ZL} and $T_{IN}(0)$ ($N = 1000$, $L = 2$, $\gamma = -12$ dB, $N_s = 3$, $\omega_0 = 0$).

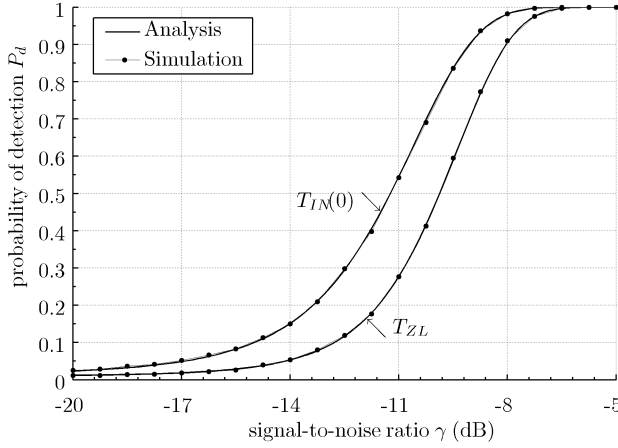


Fig. 3. CFAR P_d vs. γ ($P_{fa} = 0.01$, $N = 1000$, $L = 2$, $N_s = 3$, $\omega_0 = 0$).

frequency offset $\omega_0 = 0.1\pi$. Fig. 4 shows the probability of detection of the proposed algorithm as a function of the scanning frequency offset ω . The Zeng-Liang approach which does not use ω is also shown as a reference.

As designed, the detection probability $P_{d,IN}$ peaks at $\omega = \omega_0$ and rolls off as ω moves away from ω_0 . The detection rate remains high over the signal band (roughly $2\pi/N_s$ rad around ω_0), indicating that the proposed method can identify the spectral location of s_n . This is an advantage over the Zeng-Liang algorithm which, by construction, cannot detect the location of s_n over the system bandwidth.

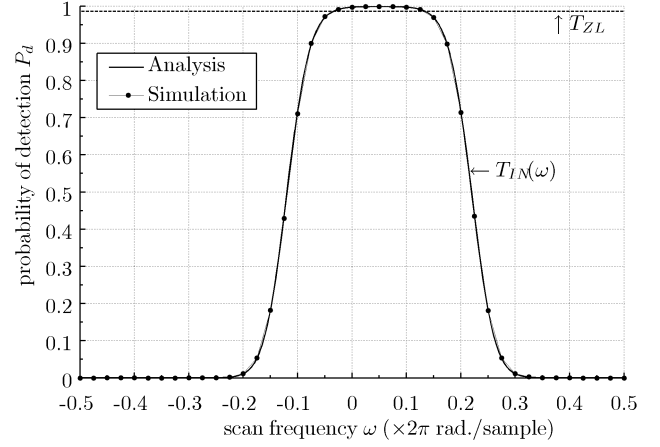


Fig. 4. CFAR $P_{d,IN}$ vs. ω ($P_{fa} = 0.01$, $N = 1000$, $L = 2$, $\gamma = -8$ dB, $N_s = 3$, $\omega_0 = 0.1\pi$).

VI. CONCLUSION

Autocorrelation-based spectral sensing techniques are investigated and analyzed. A new technique using the sample autocorrelation function of the received signal is introduced and its performance is evaluated theoretically. Furthermore, the analysis is extended to obtain the performance of the covariance-based detector in [1]. The performance of the two detectors are also obtained through Monte Carlo simulations. The results show a very good match between the theoretical and simulation results. This verifies the accuracy of our asymptotic analysis. Furthermore the results show that the proposed method outperforms the covariance-based method in [1].

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